Chapter 4
Performance Analysis of Interference Localization Based on Doppler Frequency Shift of a Single Satellite

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Abstract Two interference localization methods for a single satellite based on Doppler frequency are introduced in this paper, with the relationship between the localization error, frequency measurement accuracy and ephemeris accuracy derived. By using ephemeris data of synchronous orbit satellites, the interference localization performance is analyzed in detail, resulting in the quantitative estimation about the influence on positioning of each factor. According to the results, the target references of all factors are presented to achieve hundred-kilometer-level positioning error, as well as some considerations in implementation.

Keywords Doppler frequency · GEO satellite · IGEO satellite · Interference localization

4.1 Introduction

With the rapid development of satellite communication, the interference on ground becomes a threat to the normal operation of the satellite communication system, which makes the interference source localization necessary. Although the most effective method used commonly is the geolocation method using TDOA & FDOA, the requirements in the localization operation limits its application. If the interference can be detected only in the signal transmitted by the interfered satellite, and there is no partner satellite with the same polarization mode, it is difficult to use this method for interference source localization. At present, the synchronous orbit satellite is the most commonly type used in satellite communication system, whose larger height (about 36,000 km) and severe running

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environment make the study on the corresponding interference localization technology very necessary.

Interference localization technology based on the Doppler frequency shift [1, 2] is an effective solution for interference localization aiming to a single satellite. Two localization methods based on Doppler frequency shift of a single satellite are presented in this paper, as well as in-depth study on interference localization performance of GEO satellites and IGEO satellites [3]. Through the positioning precision formula derivation and data processing, quantitative analysis on the relationship between the positioning error and each factor of the positioning accuracy is presented, resulting in some related conclusion on interference localization based on Doppler shift of a single synchronous orbit satellite.

4.2 The Interference Localization Principle Based on Doppler Frequency Shift of a Single Satellite

Due to gravity, the moon's gravity and atmospheric drag effect, a standard signal received in ground via a synchronous orbit satellite transmission will have a Doppler offset [4], which is just the frequency deviation of the signal received in ground from that emitted by the satellite:

$$f_d = f_r - f_0 = \frac{f_0}{C} \cdot \frac{\vec{v} \cdot \vec{r}}{\|r\|} \tag{4.1}$$

where f_0 is the emitting frequency, f_r is the frequency received, C is the velocity of light, $\vec{v}$ is the relative velocity of the satellite to the stationary satellite orbit, $\frac{\vec{r}}{\|r\|}$ is the unit vector in the connection direction between the receiving ground station and the satellite.

Therefore, in a certain coordinate system and a certain moment, assuming that the coordinate of the interference source is $\vec{x}_I = (x, y, z)$, the coordinate of the satellite is $\vec{x}_S = (x_s, y_s, z_s)$, the frequency of interference source is f_I, the local oscillator frequency of the satellite transponder is f_{T0}, the frequency received in ground is defined as:

$$f_{Er} = (f_I \cdot (1 + \frac{\vec{v} \cdot (\vec{x}_I - \vec{x}_S)}{\|\vec{x}_I - \vec{x}_S\|}) + f_{T0}) \cdot (1 + \frac{\vec{v} \cdot (\vec{x}_E - \vec{x}_S)}{\|\vec{x}_E - \vec{x}_S\|}) \tag{4.2}$$

while the corresponding Doppler frequency is defined as:

$$f_d = f_{Er} - f_I \tag{4.3}$$

So, it is possible to achieve the position of interference source from the relationship equation between interference source coordinate and Doppler frequency.

4.2.1 Positioning Model Based on Doppler Frequency Offset

When a motionless jammer in ground transmits a signal with stable frequency continuously, aiming at a satellite, it is possible to accomplish the interference localization using the frequencies of signals received in ground at 4 moments. Positioning model based on received frequencies at multiple moments is as follows:

$$\begin{cases} f_{Er}^{i} = \left(f_I \cdot \left(1 + \vec{v}_i \cdot \dfrac{\vec{r}_i}{\|\vec{r}_i\|}\right) + f_{T0}\right) \cdot \left(1 + \vec{v}_i \cdot \dfrac{\vec{r}_{Ei}}{\|\vec{r}_{Ei}\|}\right) \\ \vec{r}_i = \vec{x}_I - \vec{x}_{Si}, \quad \vec{r}_{Ei} = \vec{x}_E - \vec{x}_{Si} \ (i = 1, 2, 3, 4\ldots) \end{cases} \tag{4.4}$$

For a synchronous orbit satellite, position variation of the satellite is so small that makes the assumption possible that position and velocity of the satellite are constants during the signal transmission from a jammer to the satellite, then to the earth station. Then, a solution for the position and frequency of the jammer is presented in Eq. (4.4), as long as frequencies of more than 4 moments are acquired.

4.2.2 Positioning Model Based on Doppler Frequency Difference

To decrease the effect of interference signal frequency error and satellite transponder local oscillator frequency error, Doppler frequency difference between two moments can be used to interference localization and the corresponding model is as follows:

$$\begin{cases} \Delta f_d^{i} = f_{Er}^{i} - f_{Er}^{i+1} \\ f_{Er}^{i} = (f_I \cdot (1 + \vec{v}_i \cdot \dfrac{\vec{r}_i}{\|\vec{r}_i\|}) + f_{T0}) \cdot (1 + \vec{v}_i \cdot \dfrac{\vec{r}_{Ei}}{\|\vec{r}_{Ei}\|}) \\ \vec{r}_i = \vec{x}_I - \vec{x}_{Si}, \quad \vec{r}_{Ei} = \vec{x}_E - \vec{x}_{Si} \ (i = 1, 2, 3, 4\ldots) \end{cases} \tag{4.5}$$

From Eq. (4.5) above, if the Doppler frequency difference used for interference localization, then frequencies received in ground at more than 4 moments are needed.

4.3 Analysis of Positioning Error

Interference localization method based on the Doppler frequency shift achieved the positioning solution, by establishing the relationship between the observation vector and target position. Make Z for the observation vector, which is expressed as $Z = [f_{Er}^1, f_{Er}^2, \ldots, f_{Er}^Q]^T$ or $Z = [\Delta f_d^1, \Delta f_d^2, \ldots, \Delta f_d^Q]^T$, where Q is the number of observing moments. Then the left of the localization equation is the function of the measurements, as the right the function of satellite positions and velocities at different moments, the local oscillator frequency of satellite transponder, frequency and position of the jammer, which is supposed as $F(X, X_1, \ldots X_M, V_1, \ldots V_M, f_I, f_{T0})$, where M is the number of all satellite moments used. So, Eqs. (4.4, 4.5) can be expressed as follows:

$$G(Z) = F(X, X_1, X_M, V_1, \ldots V_M, f_I, f_{T0}) \tag{4.6}$$

To calculate the differential of Eq. (4.6), make the error instead of the differential [5, 6], when the error is small, then there is

$$H_Z d_Z = HdX + \sum_{i=1}^{M} H_i dX_i + \sum_{i=1}^{M} H_{V_i} dV_i + H_{f_I} df_I + H_{f_{T_0}} df_{T0} \tag{4.7}$$

where $dX = [dx\ dy\ dz]^T$ representing the interference source localization error vector, $dX_i = [dx_i\ dy_i\ dz_i]^T$, $VdV_i = [dv_{xi}\ dv_{yi}\ dv_{zi}]^T$ $(i = 1, \ldots M)$ representing satellite position and velocity error vector at different moments, df_I representing the jammer frequency precision with df_{T0} the local oscillator frequency precision of satellite transponder and dZ the observation error vector, i.e. frequency measurement error (frequency or frequency difference measurement error).

$$H_Z = \frac{\partial G}{\partial Z} \quad H_X = \frac{\partial F}{\partial X} \quad H_{X_i} = \frac{\partial F}{\partial X_i}$$
$$H_{V_i} = \frac{\partial F}{\partial V_i} \quad H_{f_I} = \frac{\partial F}{\partial f_I} \quad H_{f_{T0}} = \frac{\partial F}{\partial f_{T0}}$$

Then, interference localization error of the can be expressed as follows:

$$\begin{aligned} dX = (H^T H)^{-1} H^T (H_Z dZ - \sum_{i=1}^{M} H_i dX_i \\ - \sum_{i=1}^{M} H_{V_i} dV_i - H_{f_I} df_I - H_{f_{T0}} df_{T0}) \end{aligned} \tag{4.8}$$

Assuming that the random measurement errors of all parameters are independent and conform to Gauss distribution with zero mean, as well as the satellite position and velocity measurement error component has the same standard difference, i.e., $df_{Er} \sim N(0, \sigma_f^2)$, $d\Delta f_d \sim N(0, \sigma_{\Delta f_d}^2)$, $dv_{xi}, dv_{yi}, dv_{zi} \sim N(0, \sigma_v^2)$, $dx_i, dy_i, dz_i \sim N(0, \sigma_s^2)$, $df_I \sim N(0, \sigma_{f_I}^2)$, $df_{T0} \sim N(0, \sigma_{f_{T0}}^2)$, the localization error covariance matrix P_X can be expressed as follows:

$$
\begin{aligned}
P_X &= E(dX \cdot dX^T) = (H^T H)^{-1} H^T \cdot \big[H_Z \sigma_Z^2 H_Z^T \\
&\quad + \sum_{i=1}^{M} H_{X_i} \sigma_S^2 H_{X_i}^T + \sum_{i=1}^{M} H_{V_i} \sigma_v^2 H_{V_i}^T + H_{f_I} \sigma_{f_I}^2 H_{f_I}^T \\
&\quad + H_{f_{T_0}} \sigma_{f_{T_0}}^2 H_{f_{T_0}}^T \big] \cdot H (H^T H)^{-1}
\end{aligned}
\tag{4.9}
$$

where, $\sigma_Z = \sigma_f$ or $\sigma_Z = \sigma_{\Delta f_d}$.

Then the root-mean-square error of Interference source position is expressed as:

$$
\sigma_x = \sqrt{trace(P_X)} \tag{4.10}
$$

where $trace(P_X)$ is the trace of covariance matrix P_X. Usually, Positioning accuracy is expressed by circular error probability (CEP), and the relationships between equivalent error radius R and root mean square error in 50 % probability condition are as follows [7]:

$$
\text{3D: } R = 1.5381\sigma_x \tag{4.11}
$$

$$
\text{2D: } R = 1.1774\sigma_x \tag{4.12}
$$

4.4 Performance Analysis and Conclusion

4.4.1 Experiments

The localization performance of two methods in this paper is tested by experiments, using ephemeris data of synchronous orbit satellites in a satellite system. According to the satellite simulation data, variation of Doppler frequency shift in a running cycle of a synchronous orbit satellite is shown in Fig. 4.1. For the effective analysis of the influence on localization performance of various factors, information of satellite positions and velocities at five moments marked by diamond in Fig. 4.1 is selected to establish the localization equations, when the Doppler frequency shifts are of the peak value or zero. According to the analysis result mentioned in segment 3, the corresponding positioning error formula is derived with the algorithm programmed based on the localization equations, achieving the positioning error results under various factors of different value.

4.4.2 Test Results and Error Analysis

In order to analyze the effect of various factors on localization exclusively, errors of other factors are supposed to be zero, when analyzing the effects of non-frequency measurement error, and the frequency measurement error is selected to

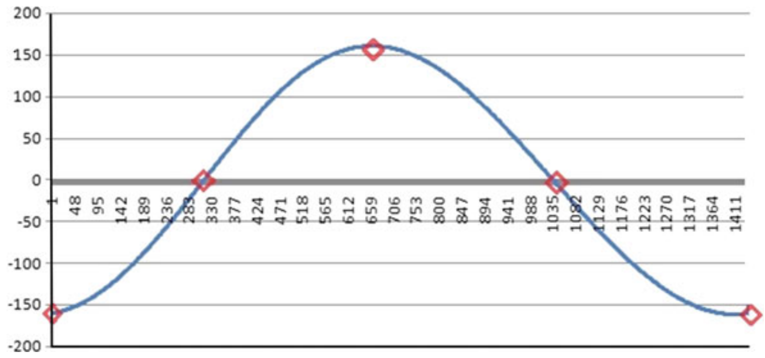

Fig. 4.1 *Curve* variation of Doppler frequency shift

ensure a 100 km-level positioning error. Specific data are presented in Tables 4.1 and 4.2.

Results of error analysis are as follows:

(1) To achieve the same level of positioning error, frequency measurement accuracy demand of GEO is different from IGEO. Positioning error increases in almost the same quantitative level as the frequency measurement error increasing. In condition of the same frequency measurement accuracy, positioning performance difference between two methods for GEO is more distinct than that for IGEO. According to data in Table 4.1, the frequency measurement accuracy demand of GEO and IGEO are respectively 10^{-9} Hz and 10^{-5} Hz for a hundred-kilometer-level positioning error.

(2) The effect of satellite position error is small when that is less than 50 m for GEO, but for IGEO, satellite position error has almost no effect on the final positioning. So, in these two cases, influence of satellite position can be ignored. Compared with satellite position, effect of satellite velocity on the positioning is more obvious. It is seen that from data in Table 4.2, satellite velocity accuracy demand of GEO is different from IGEO with the positioning

Table 4.1 Variation of positioning error along with frequency measurement accuracy ($\sigma_S = \sigma_v = \sigma_{f_l} = \sigma_{f_{T_0}} = 0$)

Frequency measurement accuracy (Hz)	Method based on Doppler frequency offset (km)		Method based on Doppler frequency difference (km)	
	GEO	IGEO	GEO	IGEO
10^{-9}	*37.0157*	0.0088	*61.5080*	0.0083
10^{-5}	3.7016×10^5	*88.1309*	6.1508×10^5	*82.9095*
10^{-3}	3.7016×10^7	8.8131×10^3	6.1508×10^7	8.2909×10^3
10^{-2}	3.7016×10^8	8.8131×10^4	6.1508×10^8	8.2909×10^4
0.1	3.7016×10^9	8.8131×10^5	6.1508×10^9	8.2909×10^5
1	3.7016×10^{10}	8.8131×10^6	6.1508×10^{10}	8.2909×10^6

Table 4.2 Variation of positioning error along with other factors except for frequency measurement accuracy (GEO: $\sigma_Z = 10^{-9}$, IGEO: $\sigma_Z = 10^{-5}$)

Factor/unit	Factor error (accuracy/precision)	Method based on Doppler frequency offset (km)		Method based on Doppler frequency difference (km)	
		GEO	IGEO	GEO	IGEO
Satellite position (m)	20	43.6000	88.1312	63.6541	82.9123
	50	68.4664	88.1327	73.9056	82.9166
	100	120.9963	88.1597	102.4619	82.9541
	500	577.1643	88.3105	414.3225	83.1880
	1,000	1,152.5468	88.8472	821.7682	84.0181
Satellite velocity (m/s)	10^{-6}	37.0221	88.1309	61.5182	82.9095
	10^{-2}	6.8678×10^3	88.1461	1.1217×10^4	82.9374
	10^{-1}	6.8678×10^4	89.6350	1.1217×10^5	85.6516
	1	6.8678×10^5	185.7528	1.1217×10^6	230.4232
	10	6.8678×10^6	1637.5196	1.1217×10^7	2,151.5025
Interference frequency (Hz)	10^{-8}	417.2249	88.6435	61.5080	82.9095
	10^{-5}	4.1558×10^5	129.7245	61.5080	82.9095
	10^{-3}	4.1558×10^7	9.5195×10^3	61.5082	82.9095
	0.1	4.1558×10^9	9.5195×10^5	63.2231	82.9525
	1	4.1558×10^{10}	9.5195×10^6	158.6669	87.1103
Oscillator frequency (Hz)	10^{-9}	55.6527	88.1309	61.5080	82.9095
	10^{-5}	4.1558×10^5	129.7245	61.5080	82.9095
	10^{-3}	4.1558×10^7	9.5195×10^3	61.5081	82.9095
	0.1	4.1558×10^9	9.5195×10^5	61.9412	82.9203
	1	4.1558×10^{10}	9.5195×10^6	95.5575	83.9794

error change trend similar, which is that positioning error increases suddenly when the satellite velocity error reaches a certain level.

(3) Demand on frequency precision of interference source is very high in localization method based on Doppler frequency shift, almost equaling to that of frequency measurement accuracy. In the method based on Doppler frequency difference, interference source emission frequency precision requirements are relatively several orders of magnitude lower. However, there is a great degree of deterioration in positioning accuracy for both two methods, when the emission frequency error reaches a certain magnitude. Fluctuations of local oscillator frequency of satellite transponder have an effect on the frequency received in ground, whose influence is similar to that of interference source emission frequency.

4.4.3 Performance Aanalysis Conclusions

As mentioned above all, in the interference localization methods based on Doppler frequency shift of a single satellite, main factors effecting the final positioning include frequency received in ground, satellite position, satellite velocity, interference emission frequency and local oscillator frequency of satellite transponder. Precision requirements of each factor for a 100 km-level of localization accuracy are shown in Table 4.3. Usually, crystal oscillator frequency stability and accuracy of the satellite transponder is better than 10^{-9}, so effect of local oscillator frequency can be ignored, and effect of the satellite velocity may be reduced by ephemeris correction algorithm mentioned in paper [8, 9]. Therefore, more attention should be paid to the receiving frequency measurement accuracy in ground and inference emission frequency precision for the performance analysis interference localization based on Doppler frequency shift of a single satellite.

From the above analysis, conclusions can be drawn: In condition of the same frequency measurement accuracy, positioning error of the method based on Doppler frequency difference is slightly higher than method based on frequency offset, whose requirement of interference source emission frequency precision is much lower. In fact, frequency accuracy of removable interference equipment is usually below the level of 10^{-8} Hz, in which case the method based on Doppler frequency difference is obviously easier for realization. In practical application, effect of interference frequency stability should be considered, for its result frequency deviation will be added to the frequency received in ground directly, and be regarded as a part of Doppler frequency shift in positioning solution, which results in positioning error. Therefore, it is suggested that time domain transient measurement mode should be used in the measurement of frequency received in ground and the frequency measurement accuracy be lower than stability of interference emission frequency.

Targets of factors presented in Table 4.3 are estimated using satellite data of a whole running cycle, and some adjustment should be done in practical application based on targets in Table 4.3 according to the actual duration of disturbance, with an enhancement under the duration less than 24 h.

Table 4.3 Target references of each factor for 100 km-level positioning error

Target	Frequency measurement error (m)	Satellite position error (m)	Satellite velocity error (m/s)	Interference frequency error (Hz)	Interference frequency error (Hz)
GEO: method based on frequency offset	$\leq 10^{-9}$	≤ 50	$\leq 10^{-4}$	$\leq 10^{-9}$	$\leq 10^{-9}$
IGEO: method based on frequency offset	$\leq 10^{-5}$	None	≤ 0.1	$\leq 10^{-5}$	$\leq 10^{-5}$
GEO: method based on frequency difference	$\leq 10^{-9}$	≤ 100	$\leq 10^{-4}$	≤ 0.1	≤ 1
IGEO: method based on frequency difference	$\leq 10^{-5}$	None	≤ 0.1	≤ 1	≤ 1

4.5 Ending

Aiming at the interference localization problem of a single synchronous-orbit satellite, theoretical and data analysis on positioning performance of two methods based on Doppler frequency shift are discussed in this paper. Based on the particular analysis on influences of all the affecting factors, reference target of each factor is given for a 100 km-level positioning accuracy, which provides important reference data for the studies on interference localization of a single synchronous-orbit satellite in communication systems.

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